LECȚIA 18 ① 18.05.

CLASA XI C PROF. SCHNAKOVSZKI C.

REGULILE LUI L'HOSPITAL

Cele două reguli ale lui L'Hospital se pot aplica pentru a găsi limitele în cazurile de nedeterminare $\frac{0}{0}$ sau $\frac{\infty}{\infty}$, cu ajutorul derivatelor.

TEOREMA 1: (Prima regulă a lui L'Hospital pentru cazul $\boxed{\frac{0}{0}}$).

Fie funcțiile $f, g : I \to \mathbb{R}$, I interval și x_0 un punct de acumulare al acestuia.

Dacă: a) f și g sunt derivabile pe $I \setminus \{x_0\}$

b) $\lim\limits_{x \to x_0} f(x) = \lim\limits_{x \to x_0} g(x) = 0$

c) $g'(x) \neq 0$ pentru $(\forall) x \in I \setminus \{x_0\}$

d) există limita $\lim\limits_{x \to x_0} \frac{f'(x)}{g'(x)} \in \overline{\mathbb{R}}$,

atunci funcția $\frac{f}{g}$ are limită în x_0 și

$$\boxed{\lim_{x \to x_0} \frac{f(x)}{g(x)} = \lim_{x \to x_0} \frac{f'(x)}{g'(x)}}.$$

Ex: Să se calculeze limitele:

a) $\lim\limits_{x\to 1} \dfrac{x^5-5x^4+4}{x^3-3x^2+2} = \left(\dfrac{0}{0}\right)$

Funcțiile $f(x) = x^5-5x^4+4$ sunt derivabile pe $\mathbb{R} \Rightarrow$
$g(x) = x^3-3x^2+2$

$\Rightarrow$ aplicăm Regula lui L'Hospital:

$$\lim_{x\to 1} \frac{x^5-5x^4+4}{x^3-3x^2+2} = \lim_{x\to 1} \frac{(x^5-5x^4+4)'}{(x^3-3x^2+2)'} = \lim_{x\to 1} \frac{5x^4-20x^3}{3x^2-6x} =$$

$$= \frac{5-20}{3-6} = \frac{-15}{-3} = 5$$

b) $\lim\limits_{x\to 0} \dfrac{e^x-e^{2x}}{\ln(x+1)} = \left(\dfrac{0}{0}\right) \Rightarrow \lim\limits_{x\to 0} \dfrac{(e^x-e^{2x})'}{(\ln(x+1))'} =$

$$= \lim_{x\to 0} \frac{e^x-2\cdot e^{2x}}{\frac{1}{x+1}} = \frac{e^0-2\cdot e^0}{\frac{1}{1}} = \frac{1-2}{1} = -1$$

c) $\lim\limits_{x\to 2} \dfrac{\sqrt{x+2}-2}{x^2-4} = \left(\dfrac{0}{0}\right) = \lim\limits_{x\to 2} \dfrac{(\sqrt{x+2}-2)'}{(x^2-4)'} = \lim\limits_{x\to 2} \dfrac{\frac{1}{2\sqrt{x+2}}}{2x} =$

$$= \lim_{x\to 2} \frac{1}{4x\sqrt{x+2}} = \frac{1}{8\sqrt{4}} = \frac{1}{8\cdot 2} = \frac{1}{16}.$$

d) $\lim\limits_{x\to 0} \dfrac{x-\ln(x+1)}{x^2} = \left(\dfrac{0}{0}\right) = \lim\limits_{x\to 0} \dfrac{[x-\ln(x+1)]'}{(x^2)'} =$

$= \lim [illegible] \; 1 - \dfrac{1}{x+1} \; [illegible] = \left(\dfrac{0}{[illegible]}\right) = \lim \dfrac{1' - \left(\frac{1}{x+1}\right)'}{[illegible]} = \lim\limits_{x\to 0} \dfrac{\frac{1}{(x+1)^2}}{2} = \dfrac{1}{2}$

e) $\lim\limits_{x\to 0}\dfrac{x-\sin x}{x^3}=\left(\dfrac{0}{0}\right)=\lim\limits_{x\to 0}\dfrac{(x-\sin x)'}{(x^3)'}=\lim\limits_{x\to 0}\dfrac{1-\cos x}{3x^2}=\left(\dfrac{0}{0}\right)=$

$=\lim\limits_{x\to 0}\dfrac{(1-\cos x)'}{(3x^2)'}=\lim\limits_{x\to 0}\dfrac{\sin x}{6x}=\lim\limits_{x\to 0}\dfrac{\sin x}{x}\cdot\dfrac{1}{6}=1\cdot\dfrac{1}{6}=\dfrac{1}{6}$

f) $\lim\limits_{x\to -\frac{1}{2}}\dfrac{\arcsin(2x+1)}{4x^2-1}=\left(\dfrac{0}{0}\right)=\lim\limits_{x\to -\frac{1}{2}}\dfrac{\frac{2}{\sqrt{1-(2x+1)^2}}}{8x}=\dfrac{\frac{2}{\sqrt{1-(-1+1)^2}}}{8\cdot\left(-\frac{1}{2}\right)}$

$=\dfrac{2}{-4}=-\dfrac{1}{2}$

Sau $\lim\limits_{x\to -\frac{1}{2}}\dfrac{\arcsin(2x+1)}{(2x+1)}\cdot\dfrac{1}{2x-1}=$

$=\underbrace{\lim\limits_{x\to -\frac{1}{2}}\dfrac{\arcsin(2x+1)}{2x+1}}_{1}\cdot\lim\limits_{x\to -\frac{1}{2}}\dfrac{1}{2x-1}=1\cdot\dfrac{1}{2\cdot\left(-\frac{1}{2}\right)-1}=$

$=1\cdot\dfrac{1}{-1-1}=-\dfrac{1}{2}.$

g) $\lim\limits_{x\to 0}\dfrac{\cos 2x-\cos 3x}{\sin 5x\,\sin 8x}=\left(\dfrac{0}{0}\right)=\lim\limits_{x\to 0}\dfrac{(\cos 2x-\cos 3x)'}{(\sin 5x\cdot\sin 8x)'}=$

$=\lim\limits_{x\to 0}\dfrac{-2\sin 2x+3\sin 3x}{5\cos 5x\cdot\sin 8x+8\sin 5x\cos 8x}=\left(\dfrac{0}{0}\right)=$

$\cos=0$
$\sin 0=0$

$=\lim\limits_{x\to 0}\dfrac{-4\cos 2x+9\cos 3x}{-25\sin 5x\cdot\sin 8x+40\cos 5x\cos 8x+40\cos 5x\cos 8x-64\sin 5x\cdot\sin 8x}=$

$=\dfrac{-4\cos 0+9\cos 0}{-25\cdot\sin 0\cdot\sin 0+40\cos 0\cdot\cos 0+40\cdot\cos 0\cdot\cos 0-64\cdot\sin 0\cdot\sin 0}=$

h) $$\lim_{x\to 0} \frac{1+x\sin x - \cos 2x}{\sin^2 x} \left(\frac{0}{0}\right) = \lim_{x\to 0} \frac{(1+x\sin x - \cos 2x)'}{(\sin^2 x)'} =$$

$$= \lim_{x\to 0} \frac{0 + x'\cdot \sin x + x(\sin x)' - (\cos 2x)'}{2\sin x \cdot (\sin x)'} =$$

$$= \lim_{x\to 0} \frac{1\cdot \sin x + x\cdot \cos x + \sin 2x \cdot 2}{2\sin x \cdot \cos x} = \left(\frac{0}{0}\right) =$$

$$= \lim_{x\to 0} \frac{\cos x + 1\cdot(+\cos x) + x\cdot(-\sin x) + 4\cos 2x}{2(\cos x\cdot \cos x - \sin x \cdot \sin x)} =$$

$$= \lim_{x\to 0} \frac{2\cos x - x\sin x + 4\cos 2x}{2\cdot(\cos^2 x - \sin^2 x)} = \frac{2\cdot 1 - 0 + 4\cdot 1}{2(1-0)} = \frac{6}{2} = 3$$

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