LECȚIA 8: EXERCIȚII - DERIVATA DE ORDINUL II - clasa a XI-a C

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1. Să se arate că funcția:

a) $f(x)=\alpha\cdot e^{2x}+\beta\cdot e^{3x}$, $\alpha,\beta,x\in\mathbb{R}$, verifică egalitatea: $f''(x)-5\cdot f'(x)+6\cdot f(x)=0$, $x\in\mathbb{R}$.

$$f(x)=\alpha\cdot e^{2x}+\beta\cdot e^{3x}$$

$$f'(x)=(\alpha\cdot e^{2x}+\beta\cdot e^{3x})'=\alpha\cdot 2\cdot e^{2x}+\beta\cdot 3\cdot e^{3x}$$

$$f''(x)=(2\alpha\cdot e^{2x}+3\beta\cdot e^{3x})'=2\alpha\cdot 2\cdot e^{2x}+3\beta\cdot 3\cdot e^{3x}=$$
$$=4\alpha\cdot e^{2x}+9\cdot\beta\cdot e^{3x}$$

Înlocuim în rel.:

$$4\cdot\alpha\cdot e^{2x}+9\cdot\beta\cdot e^{3x}-5(2\cdot\alpha\cdot e^{2x}+3\cdot\beta\cdot e^{3x})+6(\alpha\cdot e^{2x}+\beta\cdot e^{3x})=0$$

$$\underline{4\cdot\alpha\cdot e^{2x}}+\underline{9\cdot\beta\cdot e^{3x}}\underline{-10\cdot\alpha\cdot e^{2x}}\underline{-15\cdot\beta\cdot e^{3x}}\underline{+6\alpha\cdot e^{2x}}+\underline{6\beta\cdot e^{3x}}=0$$

$0=0$ (A)

b) $f(x)=x\cdot e^{2x}$ verifică: $f''(x)-4\cdot f'(x)+4\cdot f(x)=0, x\in\mathbb{R}$

$$f'(x)=(x\cdot e^{2x})'=x'\cdot e^{2x}+x\cdot(e^{2x})'=1\cdot e^{2x}+x\cdot 2\cdot e^{2x}=$$
$$=e^{2x}+2x\cdot e^{2x}$$

$$f''(x)=(e^{2x}+2x\cdot e^{2x})'=(e^{2x})'+(2x\cdot e^{2x})'=$$
$$=2e^{2x}+(2x)'\cdot e^{2x}+2x\cdot(e^{2x})'=$$
$$=2\cdot e^{2x}+2\cdot e^{2x}+2x\cdot 2\cdot e^{2x}=$$
$$=4\cdot e^{2x}+4x\cdot e^{2x}.$$

Inlocuim in rel :=)

$4\cdot e^{2x}+4\cdot x\cdot e^{2x}-4\cdot(e^{2x}+2x\cdot e^{2x})+4\cdot x\cdot e^{2x}=0$

$\cancel{4\cdot e^{2x}}+4\cdot x\cdot e^{2x}\cancel{-4\cdot e^{2x}}-8\cdot x\cdot e^{2x}+4\cdot x\cdot e^{2x}=0$

$0=0$ (A)

c) $f(x)=\alpha\cdot e^{2x}\cdot\cos 3x+\beta\cdot e^{2x}\cdot\sin 3x$ verifică:

$f''(x)-4\cdot f'(x)+13\cdot f(x)=0,\quad x\in\mathbb{R}$ (1)

$f'(x)=(\alpha\cdot e^{2x}\cdot\cos 3x+\beta\cdot e^{2x}\cdot\sin 3x)'=$

$=(\alpha\cdot e^{2x}\cdot\cos 3x)'+(\beta\cdot e^{2x}\cdot\sin 3x)'=$

$=\alpha\cdot(e^{2x}\cdot\cos 3x)'+\beta\cdot(e^{2x}\cdot\sin 3x)'=$ (constanta trece in față)

$=\alpha\cdot\left[(e^{2x})'\cdot\cos 3x+e^{2x}\cdot(\cos 3x)'\right]+\beta\cdot\left[(e^{2x})'\sin 3x+e^{2x}\cdot(\sin 3x)'\right]$

$=\alpha\cdot\left[2\cdot e^{2x}\cdot\cos 3x+e^{2x}\cdot 3\cdot(-\sin 3x)\right]+\beta\cdot\left[2\cdot e^{2x}\cdot\sin 3x+\right.$

$\left.+e^{2x}\cdot 3\cdot\cos 3x\right]=$

$=2\cdot\alpha\cdot e^{2x}\cdot\cos 3x-3\cdot\alpha\cdot e^{2x}\cdot\sin 3x+2\cdot\beta\cdot e^{2x}\cdot\sin 3x+3\beta\cdot e^{2x}\cos 3x=$

$=(2\alpha+3\beta)\cdot e^{2x}\cdot\cos 3x+(2\beta-3\alpha)e^{2x}\cdot\sin 3x$

$f''(x)=\left[(2\alpha+3\beta)\cdot e^{2x}\cdot\cos 3x\right]'+\left[(2\beta-3\alpha)\cdot e^{2x}\cdot\sin 3x\right]'=$

$=(2\alpha+3\beta)\cdot\left[(e^{2x})'\cdot\cos 3x+e^{2x}\cdot(\cos 3x)'\right]+$

$+(2\beta-3\alpha)\cdot\left[(e^{2x})'\cdot\sin 3x+e^{2x}\cdot(\sin 3x)'\right]=$

$=(2\alpha+3\beta)(2e^{2x}\cdot\cos 3x-e^{2x}\cdot 3\cdot\sin 3x)+$

$+(2\beta-3\alpha)\left[2\cdot e^{2x}\cdot\sin 3x+e^{2x}\cdot 3\cdot\cos 3x\right]=$

$$f''(x) = \underline{(4\alpha+6\beta)\, e^{2x}\cdot\cos 3x} - (6\alpha+9\beta)\cdot e^{2x}\cdot\sin 3x +$$

$$+ (4\beta-6\alpha)e^{2x}\cdot\sin 3x + \underline{(6\beta-9\alpha)\cdot e^{2x}\cdot\cos 3x} =$$

$$= (4\alpha+6\beta+6\beta-9\alpha)\cdot e^{2x}\cdot\cos 3x + (4\beta-6\alpha-6\alpha-9\beta)\cdot$$

$$\cdot e^{2x}\cdot\sin 3x =$$

$$\boxed{f''(x) = (-5\alpha+12\beta)\cdot e^{2x}\cdot\cos 3x + (-12\alpha-5\beta)\cdot e^{2x}\cdot\sin 3x}$$

Înlocuim în rel (1):

$$(-5\alpha+12\beta)\cdot e^{2x}\cdot\cos 3x + (-12\alpha-5\beta)\cdot e^{2x}\cdot\sin 3x - 4[(2\alpha+3\beta)\cdot$$

$$\cdot e^{2x}\cdot\cos 3x + (2\beta-3\alpha)\cdot e^{2x}\cdot\sin 3x] + 13(\alpha\cdot e^{2x}\cdot\cos 3x +$$

$$+ \beta\cdot e^{2x}\cdot\sin 3x) = 0$$

$$(\cancel{-5\alpha}+\cancel{12\beta}-\cancel{8\alpha}-\cancel{12\beta}+\cancel{13\alpha})\cdot e^{2x}\cdot\cos 3x +$$

$$+(\cancel{-12\alpha}-\cancel{5\beta}-\cancel{8\beta}+\cancel{12\alpha}+\cancel{13\beta})\cdot e^{2x}\cdot\sin 3x = 0$$

$$0 = 0 \quad (A)$$

(2) Să se rezolve ecuația: $f'''(x)=0$, în cazurile:

a) $f(x) = \dfrac{\sqrt{x}}{x+1}$, $x \neq -1$.

$$f'(x) = \left(\frac{\sqrt{x}}{x+1}\right)' = \frac{(\sqrt{x})'\cdot(x+1)-\sqrt{x}(x+1)'}{(x+1)^2} =$$

$$= \frac{\frac{1}{2}\cdot x^{-\frac{1}{2}}\cdot(x+1)-\sqrt{x}\cdot 1}{(x+1)^2} = \frac{\frac{1}{2\sqrt{x}}\cdot(x+1)-\sqrt{x}^{\,(2\sqrt{x})}}{(x+1)^2} =$$

$$= \frac{\frac{x+1-2x}{2\sqrt{x}}}{(x+1)^2} = \frac{-x+1}{2\sqrt{x}(x+1)^2}$$

$$f''(x)=\left(\frac{-x+1}{2\sqrt{x}\cdot(x+1)^2}\right)'=\frac{(-x+1)'\cdot\left(2\sqrt{x}(x+1)\right)^2-(x+1)\cdot\left[2\cdot x^{\frac{1}{2}}(x+1)^2\right]'}{\left[2\sqrt{x}\cdot(x+1)^2\right]^2}=$$

$$=\frac{-1\cdot 2\sqrt{x}(x+1)^2-(x+1)\cdot 2\cdot\left[x^{\frac{1}{2}}\cdot(x^2+2x+1)\right]'}{4\cdot x\cdot(x+1)^4}=$$

$$=\frac{-2\sqrt{x}(x+1)^2-2(x+1)\cdot\left(x^{\frac{5}{2}}+2\cdot x^{\frac{3}{2}}+x^{\frac{1}{2}}\right)'}{4x\cdot(x+1)^4}=\qquad 2\frac{1}{2}=\frac{5}{2}$$

$$=\frac{-2\sqrt{x}(x+1)^{\cancel{2}}-2\cancel{(x+1)}\cdot\left(\frac{5}{2}\cdot x^{\frac{3}{2}}+\cancel{2}\cdot\frac{3}{\cancel{2}}\cdot x^{\frac{1}{2}}+\frac{1}{2}\cdot x^{-\frac{1}{2}}\right)}{4x(x+1)^{\cancel{4}\,3}}=$$

$$=\frac{-2\sqrt{x}(x+1)-\left(\cancel{2}\cdot\frac{5}{\cancel{2}}\cdot x^{\frac{3}{2}}+2\cdot 3\cdot x^{\frac{1}{2}}+\cancel{2}\cdot\frac{1}{\cancel{2}}\cdot x^{-\frac{1}{2}}\right)}{4x\cdot(x+1)^3}=$$

$$=\frac{\frac{\sqrt{x}}{}-2\sqrt{x}(x+1)-5\,\frac{\sqrt{x}}{}\sqrt{x^3}-6\,\frac{\sqrt{x}}{}\sqrt{x}-\frac{1}{\sqrt{x}}}{4x(x+1)^3}=$$

$$=\frac{\dfrac{-2x(x+1)-5x^2-6x-1}{\sqrt{x}}}{4x(x+1)^3}=\frac{-2x^2-2x-5x^2-6x-1}{4x\sqrt{x}(x+1)^3}=$$

$$=\frac{-3x^2-8x-1}{4x(\sqrt{x})(x+1)^3}$$

$$f''(x)=0\Rightarrow -3x^2-8x-1=0\,/\cdot(-1)$$

$$3x^2+8x+1=0$$

$$\Delta=64-12=52$$

$$x_{1,2}=\frac{-8\pm\sqrt{52}}{2\cdot 3}=\frac{-8\pm 2\sqrt{13}}{6}=\frac{-4\pm\sqrt{13}}{3}$$

b) $f(x)=x\ln x \quad , \quad x>0$

$$f'(x)=(x\ln x)'=x'\cdot\ln x+x\cdot(\ln x)'=$$
$$=1\cdot\ln x+\cancel{x}\cdot\frac{1}{\cancel{x}}=\ln x+1$$

$$f''(x)=(\ln x+1)'=(\ln x)'+1'=\frac{1}{x}$$

$$f''(x)=0 \Rightarrow \frac{1}{x}=0$$

Dar $x>0 \Rightarrow$ nu are sol.

c) $f(x)=(x^2-3x)(e^{2x}) \quad , \quad x\in\mathbb{R}.$

$$f'(x)=\left[(x^2-3x)\cdot e^{2x}\right]'=(x^2-3x)'\cdot e^{2x}+(x^2-3x)\cdot(e^{2x})'=$$
$$=(2x-3)\cdot e^{2x}+(x^2-3x)\cdot 2\cdot e^{2x}=$$
$$=(2x-3+2x^2-6x)\cdot e^{2x}=(2x^2-4x-3)\cdot e^{2x}$$

$$f''(x)=\left[(2x^2-4x-3)\cdot e^{2x}\right]'=(2x^2-4x-3)'\cdot e^{2x}+$$
$$+(2x^2-4x-3)\cdot(e^{2x})'=$$
$$=(4x-4)\cdot e^{2x}+(2x^2-4x-3)\cdot 2\cdot e^{2x}=$$
$$=(4x-4+4x^2-8x-6)\cdot e^{2x}=(4x^2-4x-10)\cdot e^{2x}$$

$f''(x)=0 \Rightarrow (4x^2-4x-10)\cdot e^{2x}=0 \Rightarrow 4x^2-4x-10=0 \,/:2$

$$2x^2-2x-5=0$$

$$\Delta=4+40=44$$

$$x_{1,2}=\frac{2\pm\sqrt{44}}{2\cdot 2}=\frac{2\pm 2\sqrt{11}}{4}=\frac{1\pm\sqrt{11}}{2}$$

d) $f(x) = \text{arcctg}\, x, \quad x \in \mathbb{R}.$

$$f'(x) = (\text{arcctg}\, x)' = -\frac{1}{1+x^2}$$

$$f''(x) = \left(\frac{-1}{1+x^2}\right)' = \frac{-1\cdot(1+x^2)' - (-1)'(1+x^2)}{(1+x^2)^2} = \frac{-1\cdot 2x}{(1+x^2)^2} =$$

$$= -\frac{2x}{(1+x^2)^2}$$

$f''(x) = 0$

$$\frac{-2x}{(1+x^2)^2} = 0 \Rightarrow -2x = 0 \Rightarrow x = 0.$$

TEMĂ E1, E2, E3/253 man