LECȚIA 5 CLASA a XI-a C

25.03.2020

PROF. SCHNAKOVSZKI CĂTĂLINA

OPERAȚII CU FUNCȚII DERIVABILE

Fie $f, g : D \to \mathbb{R}$ și $x_0 \in D$, astfel încât f și g derivabile în x_0. Atunci:

1) $f+g$ este derivabilă în x_0 și $(f+g)'(x_0) = f'(x_0) + g'(x_0)$

2) $f-g$ este derivabilă în x_0 și $(f-g)'(x_0) = f'(x_0) - g'(x_0)$.

3) $f \cdot g$ este derivabilă în x_0 și $(f\cdot g)'(x_0) = f'(x_0)\cdot g(x_0) + f(x_0)\cdot g'(x_0)$

4) $\frac{f}{g}$ este derivabilă în x_0 (dacă $g(x) \neq 0$) și

$$\left(\frac{f}{g}\right)'(x_0) = \frac{f'(x_0)\cdot g(x_0) - f(x_0)\cdot g'(x_0)}{g^2(x_0)}$$

În particular,

1. $(f^2)' = 2\cdot f\cdot f'$
2. $(f^n)' = n\cdot f^{n-1}\cdot f'$
3. $\left(\frac{1}{f}\right)' = -\frac{f'}{f^2}$
4. $(c\cdot f)' = c\cdot f'$ – constanta trece în fața derivatei, c = constantă.
5. $(-g)' = -g'$.

EXERCIȚII: 1) Să se calculeze derivata funcției f și apoi $f'(x_0)$:

1. $f : \mathbb{R} \to \mathbb{R}$, $f(x) = \sin x + x^3 + x$, $x_0 = 0$.

$f'(x) = (\sin x)' + (x^3)' + (x)' = \cos x + 3x^2 + 1$, $(\forall) x \in \mathbb{R}$

am aplicat deriv. sumei

$f'(0) = \cos 0 + 3\cdot 0^2 + 1 = 1 + 0 + 1 = 2.$

2) $f:(0,+\infty)\to\mathbb{R}$, $f(x)=x^4+\ln x$, $x_0=1$

$f'(x)=(x^4)'+(\ln x)'=4x^3+\frac{1}{x}$, $(\forall)x>0$

$x=1 \Rightarrow f'(1)=4\cdot 1^3+\frac{1}{1}=4+1=5.$

3) $f:(0;+\infty)\to\mathbb{R}$, $f(x)=\frac{1}{x}+\frac{1}{x^2}+3$, $x_0=2$

$f'(x)=\left(\frac{1}{x}\right)'+\left(\frac{1}{x^2}\right)'+3'=(x^{-1})'+(x^{-2})'=-1\cdot x^{-1-1}-2\cdot x^{-2-1}=-x^{-2}-2\cdot x^{-3}=$

$=-\frac{1}{x^2}-\frac{2}{x^3}.$

$f'(2)=-\frac{1}{2^2}-\frac{\cancel{2}}{2^{\cancel{3}2}}=-\frac{1}{2^2}-\frac{1}{2^2}=-\frac{2}{2^2}=-\frac{1}{2}$

4. $f:(0;+\infty)\to\mathbb{R}$, $f(x)=-3+\sqrt{x}+\sqrt[3]{x}+\sqrt[4]{x}$, $x_0=1$

$f'(x)=(-3)'+(\sqrt{x})'+(\sqrt[3]{x})'+(\sqrt[4]{x})=0+\left(x^{\frac{1}{2}}\right)'+\left(x^{\frac{1}{3}}\right)'+\left(x^{\frac{1}{4}}\right)'=$

$=\frac{1}{2}\cdot x^{\frac{1}{2}-\frac{2}{1}}+\frac{1}{3}\cdot x^{\frac{1}{3}-\frac{3}{1}}+\frac{1}{4}\cdot x^{\frac{1}{4}-\frac{4}{1}}=$

$=\frac{1}{2}\cdot x^{-\frac{1}{2}}+\frac{1}{3}\cdot x^{-\frac{2}{3}}+\frac{1}{4}\cdot x^{-\frac{3}{4}}=$

$=\frac{1}{2\cdot x^{\frac{1}{2}}}+\frac{1}{3\cdot x^{\frac{2}{3}}}+\frac{1}{4\cdot x^{\frac{3}{4}}}=$

$=\frac{1}{2\sqrt{x}}+\frac{1}{3\sqrt[3]{x^2}}+\frac{1}{4\cdot\sqrt[4]{x^3}}$, $(\forall)x\neq 0$

$f'(1)=\frac{1}{2\sqrt{1}}+\frac{1}{3\sqrt[3]{1^2}}+\frac{1}{4\sqrt[4]{1^3}}=\frac{^{6)}1}{2}+\frac{^{4)}1}{3}+\frac{^{3)}1}{4}=\frac{10^{12}}{12}=\frac{5}{6}.$

5. $f:(0;+\infty)\to\mathbb{R}$, $f(x)=\ln x+2^x-\sin\frac{\pi}{2}$, $x_0=1$

$f'(x)=(\ln x)'+(2^x)'-\underbrace{\left(\sin\frac{\pi}{2}\right)'}_{=\text{const.}}=\frac{1}{x}+2^x\cdot\ln 2-0=$

$=\frac{1}{x}+2^x\cdot\ln 2$, $(\forall)x>0$

$f'(1)=\frac{1}{1}+2^1\cdot\ln 2=1+2\ln 2.$

6. $f:(0;+\infty)\to\mathbb{R}$, $f(x)=\log_3 x+\lg x+\sqrt{x}-5$, $x_0=3$

$$f'(x)=(\log_3 x)'+(\log_{10} x)'+(x^{\frac{1}{2}})'-(5)'=$$

$$=\frac{1}{x\cdot\ln 3}+\frac{1}{x\cdot\ln 10}+\frac{1}{2}\cdot x^{\frac{1}{2}-1}-0=$$

$$=\frac{1}{x\ln 3}+\frac{1}{x\ln 10}+\frac{1}{2}\cdot x^{-\frac{1}{2}}=$$

$$=\frac{1}{x\ln 3}+\frac{1}{x\ln 10}+\frac{1}{2\sqrt{x}}\quad,(\forall)\,x>0$$

$$f'(3)=\frac{1}{3\ln 3}+\frac{1}{3\ln 10}+\frac{1}{2\sqrt{3}}.$$

7. $f:\mathbb{R}-\{1\}\to\mathbb{R}$, $f(x)=\frac{1}{x-1}$, $x_0=2$.

Aplicăm derivata lui $\frac{f}{g}=\frac{f'g-f\cdot g'}{g^2}$.

$$f'(x)=\frac{1'\cdot(x-1)-1\cdot(x-1)'}{(x-1)^2}=\frac{0\cdot(x-1)-1\cdot(x'-1')}{(x-1)^2}=\frac{-1\cdot(1-0)}{(x-1)^2}=-\frac{1}{(x-1)^2}$$

$x\neq 1$

$$f'(2)=-\frac{1}{(2-1)^2}=-\frac{1}{1^2}=-1.$$

8. $f:\mathbb{R}-\{1,2\}\to\mathbb{R}$, $f(x)=\frac{1}{x^2-3x+2}$, $x_0=0$

$$f'(x)=\frac{1'\cdot(x^2-3x+2)-1\cdot(x^2-3x+2)'}{(x^2-3x+2)^2}=\frac{0-1\cdot(2x-3+0)}{(x-1)^2\cdot(x-2)^2}=\frac{-2x+3}{(x-1)^2(x-2)^2}$$

$$f'(0)=\frac{-2\cdot 0+3}{(0-1)^2\cdot(0-2)^2}=\frac{3}{1\cdot 4}=\frac{3}{4}$$

9. $f:[0;+\infty)\to\mathbb{R}$, $f(x)=\frac{x\sqrt{x}}{x+1}$

$$f'(x)=\frac{(x\sqrt{x})'\cdot(x+1)-x\sqrt{x}\,(x+1)'}{(x+1)^2}=\frac{(x^1\cdot x^{\frac{1}{2}})'\cdot(x+1)-x\sqrt{x}\cdot(x'+1')}{(x+1)^2}=$$

$$=\frac{(x^{\frac{3}{2}})'(x+1)-x\sqrt{x}\,(1+0)}{(x+1)^2}=\frac{\frac{3}{2}\cdot x^{\frac{3}{2}-1}\cdot(x+1)-x\sqrt{x}}{(x+1)^2}=\frac{\frac{3}{2}\cdot\sqrt{x}(x+1)-x\sqrt{x}}{(x+1)^2}$$

10. $f:(0,+\infty)-\{e\}\to\mathbb{R},\ f(x)=\dfrac{\ln x}{\ln x-1}$

$$f'(x)=\frac{(\ln x)'(\ln x-1)-\ln x\cdot(\ln x-1)'}{(\ln x-1)^2}=\frac{\frac{1}{x}(\ln x-1)-\ln x(\ln x'-1')}{(\ln x-1)^2}=$$

$$=\frac{\frac{1}{x}(\ln x-1)-\ln x\left(\frac{1}{x}-0\right)}{(\ln x-1)^2}=\frac{\cancel{\frac{1}{x}\ln x}-\frac{1}{x}-\cancel{\frac{1}{x}\ln x}}{(\ln x-1)^2}=-\frac{1}{x(\ln x-1)^2}$$

11. $f:\mathbb{R}-\{x/\operatorname{tg}x=1\}\to\mathbb{R},\ f(x)=\dfrac{\sin x+\cos x}{\sin x-\cos x},\ x_0=\dfrac{\pi}{2}.$

$$f'(x)=\frac{(\sin x+\cos x)'\cdot(\sin x-\cos x)-(\sin x+\cos x)\cdot(\sin x-\cos x)'}{(\sin x-\cos x)^2}=$$

$$=\frac{[(\sin x)'+(\cos x)'](\sin x-\cos x)-(\sin x+\cos x)[(\sin x)'-(\cos x)']}{(\sin x-\cos x)^2}=$$

$$=\frac{(\cos x-\sin x)(\sin x-\cos x)-(\sin x+\cos x)(\cos x\overset{+}{-}(-\sin x))}{(\sin x-\cos x)^2}=$$

$$=\frac{\cancel{\cos x\cdot\sin x}-\cos^2 x-\sin^2 x+\cancel{\sin x\cdot\cos x}-\cancel{\sin x\cdot\cos x}-\sin^2 x-\cos^2 x-\cancel{\sin x\cdot\cos x}}{(\sin x-\cos x)^2}$$

$$=\frac{2\sin^2 x-2\cos^2 x}{(\sin x-\cos x)^2}=\frac{-2\cdot(\sin^2 x+\cos^2 x)}{(\sin x-\cos x)^2}=\frac{-2}{(\sin x-\cos x)^2}$$

$$f'\left(\frac{\pi}{2}\right)=\frac{-2}{\left(\sin\frac{\pi}{2}-\cos\frac{\pi}{2}\right)^2}=\frac{-2}{(1-0)^2}=\frac{-2}{1}=-2.$$

TEMĂ - manual pag 240 / E1, E2, E3, A1.

până joi.